Discrete Vortex Computation of Separated Airfoil Flow

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Abstract

LOW-SPEED unsteady separated flow may be computed from Eulerian finite-difference or -element representations of the Navier-Stokes equations or by Lagrangian vortex tracking methods. The latter have been used to model both two-dimensional separated airfoil flows at high Reynolds numbers[1,2] as well as numerous bluff-body flows.[3-5] A major difficulty with the basic vortex method is that it is inviscid; hence, separation, except at sharp edges, must be predicted by some other means. Where empirical prediction is difficult, boundary-layer calculation (viscous-inviscid matching) has been used, but it is possible to predict laminar separation too early[2] unless the discrete vortex representation is started upstream of the separation point. Viscous diffusion is usually modeled by vortex blob and random walk techniques.[6,7] The results presented in this synoptic were computed using the inviscid cloud-in-cell vortex method.[8] This is a mixed Eulerian-Lagrangian method in which descrete vortices are tracked through a grid on which the velocity field is computed by a finite-difference method. It has the advantage of faster computation when large numbers of vortices are present but a poorer resolution of the velocity field.

Contents

Cloud-in-Cell Method

In this method, the circulation of the vortices shed from the body is distributed by a weighting function (bilinear in the present calculations) onto a grid. The stream function ψ is then computed from the Poisson equation $\nabla^2\psi = -\omega$, linking ψ to the resulting vorticity distribution ω on the grid. This equation was expressed in central difference form and solved by a combination of fast Fourier transforms and Gaussian elimination. In order to avoid interpolated boundary conditions for ψ, the airfoil was transformed into a circle of radius R using the Joukowski transformation. The computation was carried out in the circle plane on a regular $n \times m$ polar mesh (r_j, θ_k), where $r_j = R(1+2\pi/m)^j$ and $\theta_k = 2k\pi/m$.

Discrete vortices were shed into the flow from the two points on the circle corresponding to the trailing edge of the airfoil and a separation point just behind the leading edge. In the present study, the flow is over an 11%-thick Joukowski airfoil at 30-deg incidence, and the leading-edge separation point was fixed empirically at 1% chord, c, from observations of the separation point on a similar airfoil tested in a wind tunnel at a Reynolds number of 8×10^5. This airfoil was fitted with endplates, and the section between the endplates, where the measurements were taken, had an aspect ratio of 3.8. Fixing the leading-edge separation point in this way clearly led to an unrepresentative flow at the start of an impulsively started flow. The Kutta-Joukowski condition was applied at both separation points in the circle plane.

Surface pressures and forces were calculated from the surface velocity q and potential ϕ using Bernoulli's theorem, ϕ being obtained by integrating q around the airfoil with appropriate discontinuities across the shedding points.

Discrete vortex calculations of separated flows tend to overpredict the suction pressure in the separated region (or base pressure), and the lift and drag forces. This was found to be the case for the present airfoil calculations. A circulation reduction technique (vortex decay with time) has been suggested to overcome this problem.[3,4] In the present computations, the strength of each vortex was varied as

$$\Gamma(\tau) = \Gamma(0)\{1 - \exp(-c/2\ U_\infty \tau)\}$$

where τ is the lifetime of the vortex, whose initial strength was $\Gamma(0)$. c was given the value of $22R$.[4]

Numerical Results

Figure 1 shows a computation of impulsively started flow past the 11%-thick Joukowski airfoil at 30-deg incidence to the stream using a 64×64 polar mesh. The figure shows the distribution of shed vorticity for the fully separated flow after a nondimensional time (based on the airfoil chord), $\hat{t} = 13.15$, computed without vortex decay. The most important feature of this result is the strong roll-up of the trailing-edge sheet above the rear upper surface. This phenomenon is quite often seen in two-dimensional separated-flow calculations, including some Navier-Stokes computations.[9] The close roll-up leads to a large suction peak in the upper surface mean pressure distribution near the trailing edge, which is not seen in the experimental measurements (Fig. 2). In this figure, the time means were taken after the initial period of starting flow was judged to have passed, until the end of each computation. The measured pressures were averaged over nondimensional times an order of magnitude greater. The time history of the lift coefficient, calculated both by integration of the surface pressure and from a momentum balance, is shown in Fig. 3. The mean value of the lift coefficient is in reasonable agreement with the measured value, but the fluctuating component is relatively large.

Vortex decay appears to have the effect of inhibiting the roll-up of the trailing-edge vortex sheet so that it now takes place further downstream and, consequently, in better agreement with flow visualization. When vortex decay is used, conservation implies an effect on the bound circulation. This may not be noticeable for many bluff bodies, but

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affects bodies, such as the airfoil, that have large mean circulation. In order to avoid this in the present calculations, the circulation removed was in effect transferred to the point at infinity. With this, the unrealistic trailing-edge suction disappeared (Fig. 2), the mean lift remained approximately the same, but the fluctuating lift was considerably reduced (Fig. 3). However, if the reduction in circulation was not transferred, the lift increased with time to unrealistic levels. The computation without vortex decay predicted a high value of 0.43 for C_{Lrms} (root-mean-square lift coefficient), reducing with vortex decay to 0.06. These values may be compared with 0.03 measured on a NACA 63018 airfoil at a Reynolds number of 5.1×10^5.[10] Other calculations have also predicted high values of C_{Lrms}.[3] Many separated flows involve three-dimensional vortex shedding, tending to give lower mean and fluctuating forces than those predicted using two-dimensional computation. Therefore, vortex reduction may partly simulate three-dimensional effects. It seems to be particularly important when the separating shear layer remains close to the surface of the body, as experienced here. The computations with between 1000 and 2000 vortices took about 1.5 CPU s per time step on a CDC 7600 computer.

Conclusions

Computation of separated flow past an airfoil at 30-deg incidence using a two-dimensional, discrete vortex cloud-in-cell method predicts too strong a roll-up of the trailing vortex sheet. This leads to an unrealistic suction peak on the rear upper surface and excessively large fluctuations in lift. Use of a vortex decay technique leads to more realistic results, provided the circulation removed is transferred so as not to affect the bound circulation.

References

[1]Katz, J., "A Discrete Vortex Method for the Non-Steady Separated Flow Over an Aerofoil," *Journal of Fluid Mechanics*, Vol. 102, 1981, p. 315.

[2]Spalart, P. R., Leonard, A., and Baganoff, D., "Numerical Simulation of Separated Flows," NASA TM 84328, 1983.

[3]Kiya, M., Arie, M., and Harigane, K., "Unsteady Separated Flow Behind a Normal Plate Calculated by a Discrete-Vortex Model," *Memoirs of the Faculty of Engineering Hokkaido University*, Vol. 15, Pt. 2, 1979, p. 211.

[4]Kiya, M., Sasaki, K., and Arie, M., "Discrete Vortex Simulation of a Turbulent Separation Bubble," *Journal of Fluid Mechanics*, Vol. 120, 1982, p. 219.

[5]Nagano, S., Naito, M., and Takata, H., "A Numerical Analysis of Two-Dimensional Flow Past Rectangular Prisms by a Discrete Vortex Method," *Journal of Fluid Mechanics*, Vol. 99, 1980, p. 225.

[6]Chorin, A. J., "Numerical Study of Slightly Viscous Flow," *Journal of Fluid Mechanics*, Vol. 57, 1973, p. 785.

[7]Cheer, A. Y., "Numerical Study of Incompressible Slightly Viscous Flow Past Blunt Bodies and Airfoils," *SIAM Journal of Computing*, Vol. 4, No. 4, 1983, p. 685.

[8]Christiansen, J. P., "Numerical Simulation of Hydrodynamics by a Method of Point Vortices," *Journal of Computational Physics*, Vol. 13, 1973, p. 363.

[9]Osswald, G. A., Ghia, K. N., and Ghia, U., "An Implicit Time-Marching Method for Studying Unsteady Flow with Massive Separation," AIAA Paper 85-1489, 1985.

[10]Lucas, C. A., "A Study of Wing Buffet," Ph.D. Thesis, Engineering Dept., Cambridge Univ., Cambridge, England, 1977.

Fig. 1 Impulsively started flow without vortex decay, $\hat{t} = 13.15$.

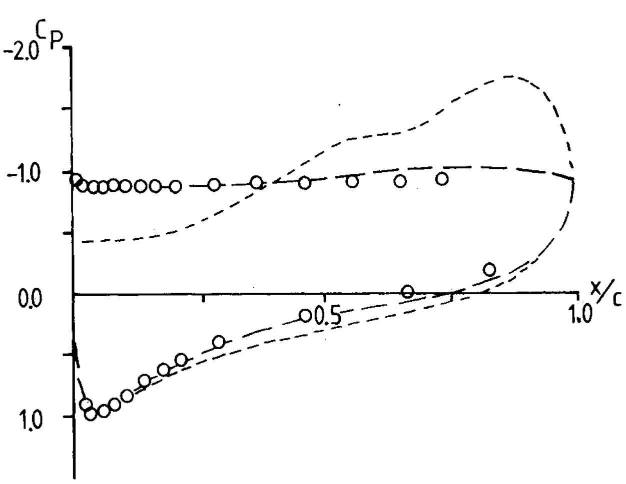

Fig. 2 Mean pressure distributions computed from $\hat{t} = 10.82$ to 12.98: - - -, without decay; — — —, with decay; ○, measured $Re = 8.0 \times 10^5$.

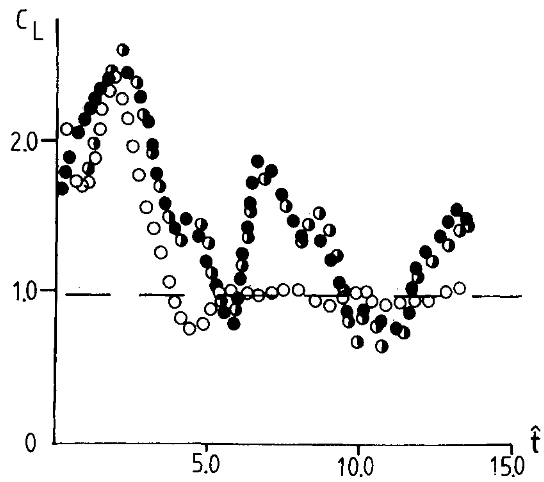

Fig. 3 Lift coefficient. Computed: ○ pressure integration, no decay; ● momentum, no decay; ○ pressure integration with decay; — — —, measured mean C_L at $Re = 8.0 \times 10^5$.